Int. J. Heat Mass Transfer. Vol. 21, pp. 335–339. Pergamon Press 1978. Printed in Great Britain

THERMAL CONVECTION IN A HORIZONTAL FLUID LAYER WITH INTERNAL HEAT SOURCES

MORTEN TVEITEREID
Department of Mechanics, University of Oslo, Norway

(*Received* 13 *December* 1976 *and in revised form* 24 *March* 1977)

Abstract—This paper is concerned with thermal convection in a horizontal fluid layer bounded below and above by two rigid planes of constant and equal temperature. The convection is generated by uniformly distributed internal heat (cool) sources. Stable hexagons are found for Rayleigh numbers up to 3.6 times the critical value, down-hexagons when the fluid is internally heated, and up-hexagons when the fluid is internally cooled. Moreover, a subcritical region, where the hexagons may exist, is also found.

NOMENCLATURE

B_{pqh}, defined by (3.5);
M, defined by (3.9);
Nu_0, Nu_1, Nusselt numbers, defined by (3.10);
Pr, $= \nu/\kappa$, Prandtl number;
Q, generated heat per unit time and unit volume;
$|Q|$, $= \sqrt{Q^2}$;
Ra, $= g\alpha|Q|d^5/64\rho_0 c_p \kappa^2 \nu$, Rayleigh number;
Ra_c, critical Rayleigh number;
T, temperature;
T_0, standard temperature;
T_s, defined by (2.5);
V, defined by (3.1);
∇, $= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$;
∇^2, $= \nabla\cdot\nabla$;
∇_1^2, $= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$;
a, wave number;
a_c, critical wave number;
c_p, specific heat at constant pressure;
d, depth of the layer;
g, acceleration due to gravity;
$\mathbf{k}$, unit vector directed upwards;
$\mathbf{v}$, $= (u, v, w)$, velocity;
t, time;
x, y, z, Cartesian coordinates.

Greek letters

α, coefficient of expansion;
δ, $= \left(\frac{\partial^2}{\partial x \partial z}, \frac{\partial^2}{\partial y \partial z}, -\nabla_1^2\right)$;
θ, temperature;
κ, thermal diffusivity;
ν, kinematic viscosity;
ρ_0, standard density;
σ, growth rate.

Superscripts

$'$, perturbation quantities;
$*$, complex conjugate quantities.

1. INTRODUCTION

THE TOPIC of this work is thermal convection in a horizontal fluid layer when heated from within by a uniform distribution of heat sources. This problem has become important in the study of motion of the Earth's mantle [1]. Internally heated convection may also be the origin of the cellular arrays in the atmosphere of Venus observed by Mariner 10 [2].

The upper and lower boundaries of the fluid layer are assumed to be perfectly conducting and maintained at constant and equal temperature, whereby we obtain an unstably stratified fluid layer above a stably stratified fluid layer. This model has been studied experimentally by Kulacki and Goldstein [3]. Applying a Mach–Zehnder interferometer, they determined the mean temperature in the fluid for various Ra varying from 0.35 Ra_c up to 675 Ra_c. In the laminar regime (from Ra_c and up to about 20 Ra_c) they observed a kind of a cellular flow. The cells seemed, however, to be neither stationary nor truly periodic. For a related model Whitehead and Chen [4] observed a flow with some similar characteristics. The supercritical motion consisted of vertical jets of cool fluid from the upper layer which plunged downward into the interior of the fluid. The jets were not arranged in an ordered lattice, but were constantly changing.

This loss of a regular pattern is in striking contrast not only to the observations for ordinary Bénard convection, but also to the results obtained in convection with internal heat sources when the lower plane is insulating [5–7]. In this last case there is no stably stratified region. It seems as the occurrence of a stably stratified region will give a tendency towards an unregular motion. In [7] we found that steady hexagons were stable for Rayleigh numbers at least up to 15 Ra_c. We shall in this paper show that with the present boundary conditions, which lead to stably as well as unstably stratified horizontal layers, steady motion is only possible for Rayleigh numbers up to 3.6 Ra_c.

2. GOVERNING EQUATIONS

We consider three-dimensional convection in an infinite horizontal fluid layer of constant depth d, bounded above and below by two rigid perfectly conducting planes maintained at constant and equal temperatures. The fluid is heated from within by a uniform distribution of heat sources.

Applying the Boussinesq approximation, the basic equations may be written

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = -\frac{1}{\rho_0}\nabla p - [1-\alpha(T-T_0)]g\mathbf{k} + \nu\nabla^2\mathbf{v}, \quad (2.1)$$

$$\nabla\cdot\mathbf{v} = 0, \quad (2.2)$$

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T = \kappa\nabla^2 T + \frac{Q}{\rho_0 c_p}, \quad (2.3)$$

with the boundary conditions

$$\mathbf{v} = 0 \quad \text{and} \quad T = 0 \quad \text{for} \quad z = 0 \quad \text{and} \quad d. \quad (2.4)$$

If the generated heat Q is sufficiently small, steady-state solutions of (2.1)–(2.4) exist of the form

$$\mathbf{v} = 0, \quad p = p_s(z), \ T = T_s(z) = \frac{Q}{2\kappa\rho_c c_p}(d-z)z. \quad (2.5)$$

If Q is sufficiently great, in the convective regime, the solutions are written in the form

$$\mathbf{v} = \mathbf{v}'(x,y,z,t), \quad p = p_s + p'(x,y,z,t), \qquad T = T_s + \theta'(x,y,z,t). \quad (2.6)$$

To get a dimensionless form of the equations we introduce d, d^2/κ, κ/d, $\kappa\nu/g\alpha d^3$ and $\kappa\nu\rho_0/d^2$ as units for length, time, velocity, temperature and pressure, respectively. Omitting the primes the equations take the form

$$Pr^{-1}\left(\frac{\partial \mathbf{v}}{\partial t} + v\cdot\nabla\mathbf{v}\right) = -\nabla p + \theta\mathbf{k} + \nabla^2\mathbf{v}, \quad (2.7)$$

$$\nabla\cdot\mathbf{v} = 0, \quad (2.8)$$

$$\frac{\partial\theta}{\partial t} + \mathbf{v}\cdot\nabla\theta = \nabla^2\theta + \frac{Q}{|Q|}32\,Ra(2z-1)w, \quad (2.9)$$

with the boundary conditions

$$\mathbf{v} = \theta = 0; \quad z = 0, 1. \quad (2.10)$$

The Rayleigh number Ra_0, defining the onset of convection, is found from the linearized version of (2.7)–(2.10). Introducing the horizontal wave number a, defined by

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = -a^2, \quad (2.12)$$

Ra_0 becomes a function of a. To obtain this function we develop the solution in a power series of z. We find that Ra_0 is independent of the sign of Q. The curve $Ra_0(a)$ is displayed in Fig. 1. The minimum value of Ra_0, Ra_c, and the corresponding value of a, a_c, are found to be

$$Ra_c = 583.2, \quad a_c = 4.00. \quad (2.13)$$

These critical numbers have also been calculated by Sparrow *et al.* [8].

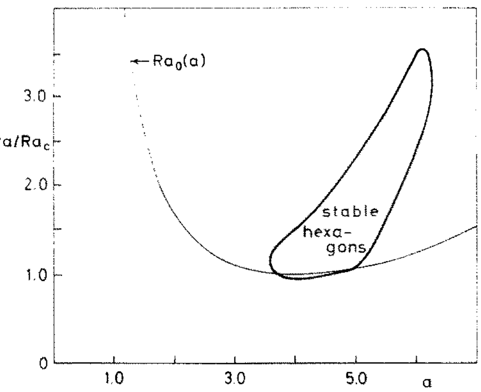

FIG. 1. The stability region.

3. STEADY SOLUTIONS

Steady solutions of the equations (2.7)–(2.10) will now be found by a numerical approach. To simplify the problem we assume that the Prandtl number is infinite, whereby (2.7) becomes linear.

Then the velocity becomes a poloidal vector (i.e. $\nabla\cdot\mathbf{v} = \mathbf{k}\cdot\nabla\times\mathbf{v} = 0$), and may appropriately be written

$$\mathbf{v} = (u, v, w) = \boldsymbol{\delta} V = \left(\frac{\partial^2}{\partial x\,\partial z}, \frac{\partial^2}{\partial y\,\partial z}, -\nabla_1^2\right)V, \quad (3.1)$$

where ∇_1^2 is the horizontal Laplacian. By eliminating the pressure term we obtain from equations (2.7)–(2.10)

$$\nabla^4 V - \theta = 0, \quad (3.2)$$

$$\nabla^2\theta - \frac{Q}{|Q|}32\,Ra(2z-1)\nabla_1^2 V - \mathbf{v}\cdot\nabla\theta = 0, \quad (3.3)$$

with the boundary conditions

$$V = \frac{\partial V}{\partial z} = \theta = 0; \quad z = 0, 1. \quad (3.4)$$

Considering periodic solutions in the x- and y-direction, we expand θ in a complete set of Fourier modes, each of them satisfying the boundary conditions

$$\theta = \sum B_{pqh}\,e^{i(pkx+qly)}\sin(h\pi z). \quad (3.5)$$

Here k and l are wave numbers in the x- and y-direction, respectively. The summation runs through all integers $-\infty < p < \infty$, $-\infty < q < \infty$, $1 \leqslant h < \infty$. To ensure (3.5) to be real, we require that $B_{pqh} = B^*_{-p-qh}$. Introducing (3.5) into (3.2) and applying the boundary conditions, we obtain

$$V = \sum B_{pqh}\,e^{i(pkx+qly)}F_h(\kappa, z), \quad (3.6)$$

where $\kappa^2 = (pk)^2 + (ql)^2$. The function $F_h(\kappa, z)$ is given in the Appendix. The unknown coefficients B_{pqh} is determined from equation (3.3) by applying a Galerkin procedure. Introducing (3.5) and (3.6) into (3.3), multiplying this by $\exp\{-i(rkx+sly)\}\sin(g\pi z)$ and averaging over the fluid layer, we obtain

$$\tfrac{1}{2}[(g\pi)^2+v^2]B_{rsg}-\frac{Q}{|Q|}32\,Rav^2\sum_h a(h,v,g)B_{rsh} - \sum_{\substack{p+t=r\\ q+u=s}}\sum_{h,f}[(ptk^2+qul^2)b(h,\kappa,f,g) + \kappa^2 c(h,\kappa,f,g)]B_{pqh}B_{tuf}=0. \quad (3.7)$$

Here $v^2=(rk)^2+(sl)^2$. The coefficients a, b and c are given in the Appendix.

It is well known, that if the coefficients in (2.7)–(2.9) are constants, the planform for moderate Rayleigh numbers will be two-dimensional rolls [9]. If the coefficients depend on z, as in the actual case, the planform may be hexagons or two-dimensional rolls. It was shown in [7] that if the linearized version of (2.7)–(2.10) is self-adjoint, the pattern will be two-dimensional rolls. If, on the other hand, the problem is not self-adjoint, we expect hexagons. In the present problem the equations are not self-adjoint, and we expect hexagons.

To the first order in the amplitude hexagons are given by the Fourier modes

$$A_{11}\cos(kx)\cos(ly)+A_{02}\cos(2ly), \quad (3.8)$$

with

$$k^2+l^2=4l^2=a^2, \quad A_{11}=\pm 2A_{02}.$$

We note that $A_{11}=0$ corresponds to two-dimensional rolls, whereas $A_{02}=0$ corresponds to a rectangular-like structure. If the first order terms are given by (3.8), it follows that the excited higher order terms will be of the form $A_{ij}\cos(ikx)\cos(jly)$ where i and j are integers such that $i+j$ are even. Restricting the solutions to this form, we may conclude that $B_{rsg}=B_{r-sg}=B_{-rsg}=B_{-r-sg}$, and that all B_{rsg} are real.

The system of differential equations (3.7) will be truncated such that the terms for which

$$g^2+\tfrac{3}{4}r^2+\tfrac{1}{4}s^2>M^2+1 \quad (3.9)$$

are neglected. Here M is an integer to be specified below. The validity of the truncation is verified by examining the values of the numbers

$$Nu_0=\frac{d}{dz}\bar{T}\Big|_{z=0}\Big/\bar{T}_{max},$$
$$Nu_1=-\frac{d}{dz}\bar{T}\Big|_{z=1}\Big/\bar{T}_{max}. \quad (3.10)$$

Here $\bar{T}$ is the horizontally averaged temperature (dimensionless) given by

$$\bar{T}=4(1-z)z+\frac{1}{8\,Ra}\sum_{g=1}^{M}B_{00g}\sin(g\pi z) \quad (3.11)$$

and $\bar{T}_{max}$ is the maximum value of $\bar{T}$. The solution is accepted to be sufficiently accurate when Nu_0 and Nu_1 vary by less than 1% as M is increased from M to $M+1$.

To find the solutions of (3.7) we apply a Newton–Raphson method. Both for Q positive and Q negative we obtain three different steady-state configurations:

(1) down-hexagons, i.e. descending flow in the center of the cells and ascending flow at the peripheries,
(2) up-hexagons and
(3) two-dimensional rolls.

In the next section we will apply a stability analysis to investigate the stability behaviour for the hexagons and the rolls.

4. STABILITY ANALYSIS

By introducing an infinitesimal perturbation $(\tilde{\theta}, \tilde{\mathbf{v}} = \boldsymbol{\delta}\,\tilde{V})$ with a time dependence of the form $\exp(\sigma t)$, the perturbation equations are written

$$\nabla^4\tilde{V}-\tilde{\theta}=0, \quad (4.1)$$

$$\nabla^2\tilde{\theta}-\frac{Q}{|Q|}32\,Ra(2z-1)\nabla_1^2\tilde{V}=\sigma\tilde{\theta}+\mathbf{v}\cdot\nabla\tilde{\theta}+\tilde{\mathbf{v}}\cdot\nabla\theta, \quad (4.2)$$

with the boundary conditions

$$\tilde{V}=\frac{\partial\tilde{V}}{\partial z}=\tilde{\theta}=0;\quad z=0,1. \quad (4.3)$$

Assuming periodical solutions in x and y, $\tilde{\theta}$ may be written

$$\tilde{\theta}=e^{i(\varepsilon kx+\delta ly)}\sum\tilde{B}_{pqh}e^{i(pkx+qly)}\sin(h\pi z), \quad (4.4)$$

where ε and δ are free parameters. It can be argued for that for hexagons, a complete stability analysis is attained when δ runs through the values from 0 to 1 and ε from 0 to $\frac{1}{3}\delta$.

From (4.1) and (4.3)

$$\tilde{V}=e^{i(\varepsilon kx+\delta ly)}\sum\tilde{B}_{pqh}e^{i(pkh+qly)}F_h(\tilde{\kappa},z) \quad (4.5)$$

where $\tilde{\kappa}=(p+\varepsilon)^2k^2+(q+\delta)^2l^2$. F_h is given in the Appendix. Multiplying (4.2) with $\exp[-i(\varepsilon kx+\delta ly)]\exp[-i(rkx+sly)\sin(g\pi z)$ and averaging over the fluid layer an infinite set of linear homogeneous equations, determining the coefficients $\tilde{B}_{rsg}$, follows. We truncate this system analogous to what was done in the previous section. The stability problem is thus reduced to the determination of σ. If all the values of σ have a negative real part, the examined configuration is stable.

5. RESULTS AND DISCUSSION

(a) Q *positive*

The results of the stability analysis are shown in Fig. 1. We find that steady convection in forms of down-hexagons exists for $Ra<3.6\,Ra_c$. For larger values of Ra the convection is unsteady which is in accordance with the observations in [3, 4]. Moreover, the wavelength of the hexagons decrease with increasing Rayleigh number, from λ_c at $Ra=Ra_c$ down to about $\frac{2}{3}\lambda_c$ at $Ra=3.6\,Ra_c$. Also a subcritical region is found, from $Ra=0.964\,Ra_c$ up to $Ra=Ra_c$. Both the up-hexagons and the rolls are unstable configurations for all values of Ra.

The numbers Nu_0 and Nu_1, defined by equation (3.10), are displayed in Fig. 2. The curves are computed by considering wave numbers in the middle of the stable region. It is seen that Nu_0 and Nu_1 vary by less

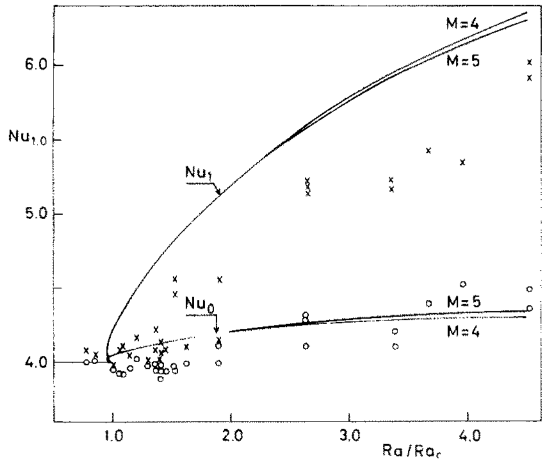

FIG. 2. The numbers Nu_0 and Nu_1 as a function of the Rayleigh number for $M = 4$ and 5. ○, x, values of Nu_0 and Nu_1, respectively, from Kulacki and Goldstein [3].

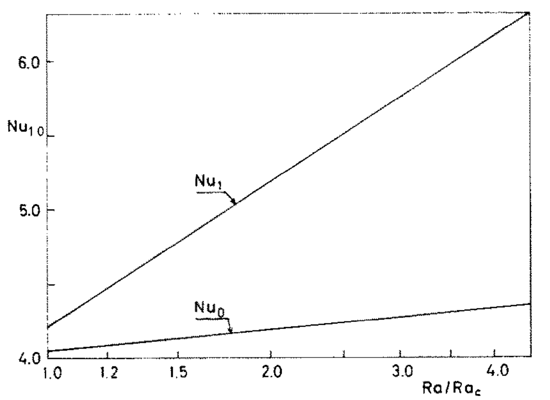

FIG. 3. Nu_0 and Nu_1 as a function of Ra, showing $Nu_0 = 4.05+0.21\log(Ra/Ra_c)$ and $Nu_1 = 4.20+1.42\log(Ra/Ra_c)$.

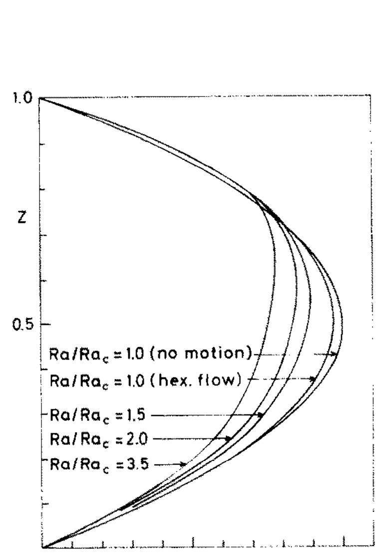

FIG. 4. The horizontally averaged temperature for different values of Ra/Ra_c.

than 1% as M increases from 4 to 5. The same numbers are also shown in Fig. 3, but with a logarithmic scale for the Ra/Ra_c-axis. To a good approximation we may write

$$\begin{aligned} Nu_0 &= 4.05+0.21\log(Ra/Ra_c), \\ Nu_1 &= 4.20+1.42\log(Ra/Ra_c). \end{aligned} \tag{5.1}$$

The horizontally averaged temperature defined by (3.11) is shown in Fig. 4. We observe that the warm central core of the layer is displayed upwards.

In Fig. 2 we also have given the experimental values of Nu_0 and Nu_1 found by Kulacki and Goldstein. They examined the convection in a fluid of aqueous silver nitrate ($Pr \approx 6$). We note that there is a marked discrepancy between the observed values of Nu_1 and the values of Nu_1 found in the present work. Between the values of Nu_0, however, there is a better agreement. The reason for this discrepancy is not clear. It may be that our solution with $Pr = \infty$ is a bad approximation to the solution with $Pr = 6$. On the other hand, the experimental values show a considerable scattering and their results are perhaps not sufficiently exact for comparison.

In the present model where the lower part of the fluid layer is stably stratified, we expected to find a region with oscillatory instability. Our computations show, however, that the marginal stable mode has a real σ.

The stability region shown in Fig. 1 was computed both for $M = 4$ and $M = 5$. We found the same stability boundaries for these two values of M, indicating that $M = 5$ defines a good truncation of the infinite system of equations.

(b) Q *negative*

In this case we find that the up-hexagons are stable in the same region as the down hexagons for Q positive (see Fig. 1). The amplitudes of these up-hexagons, B_{rsg}, multiplied with $(-1)^g$ are identical to the amplitudes of the down-hexagons we obtained for $Q > 0$. Down-hexagons and two-dimensional rolls, however, are unstable planforms when Q is negative.

6. SUMMARY AND CONCLUDING REMARKS

The main result obtained in this paper is shown in Fig. 1 where it is illustrated that hexagons are stable for Rayleigh numbers less than 3.6 Ra_c, down-hexagons when the fluid is internally heated, and up-hexagons when the fluid is internally cooled. For larger values of Ra the convection is unstable.

A comparison of the results of the present analysis with the experimental results of Kulacki and Goldstein [3] shows a rather poor agreement. This discrepancy, however, is possibly due to the fact that it is difficult to observe the convection and the convective heat transport for small supercritical Rayleigh numbers. In [3] nothing was reported about the nature of the convection for small values of Ra.

The stability boundaries obtained here are very different from those found when the lower plane is insulated, giving that the whole layer is unstably stratified [7]. It seems as the occurrence of two layers, one unstably stratified and one stably stratified changes the character of the motion thoroughly.

Acknowledgement—The author is grateful to Professor E. Palm for many valuable discussions during the preparation of this paper.

REFERENCES

1. D. P. McKenzie, J. M. Roberts and N. O. Weiss, Convection in the earth's mantle: towards a numerical simulation, *J. Fluid Mech.* **62**, 465 (1974).
2. D. J. Tritton, Internally heated convection in the atmosphere of Venus and in the laboratory, *Nature, Lond.* **257**, 110 (1975).
3. F. A. Kulacki and R. J. Goldstein, Thermal convection in a horizontal fluid layer with uniform volumetric energy sources, *J. Fluid Mech.* **55**, 271 (1972).
4. J. A. Whitehead and M. M. Chen, Thermal instability and convection of a thin fluid layer bounded by a stratified region, *J. Fluid Mech.* **40**, 549 (1970).
5. D. J. Tritton and M. N. Zarraga, Convection in horizontal layers with internal heat generation. Experiments, *J. Fluid Mech.* **30**, 21 (1967).
6. E. W. Schwiderski and H. J. A. Schwab, Convection experiments with electrolytically heated fluid layers, *J. Fluid Mech.* **48**, 703 (1971).
7. M. Tveitereid and E. Palm, Convection due to internal heat sources, *J. Fluid Mech.* **76**, 481 (1976).
8. E. M. Sparrow, R. J. Goldstein and V. K. Jonsson, Thermal instability in a horizontal fluid layer: effect of boundary conditions and non-linear temperature profile, *J. Fluid Mech.* **18**, 513 (1964).
9. A. Schlüter, D. Lortz and F. H. Busse, On the stability of steady finite amplitude convection, *J. Fluid Mech.* **23**, 129 (1965).

APPENDIX

Definition of the Function $F_h(\kappa, z)$

From (3.2)–(3.6) we obtain

$$\left(\frac{d^2}{dz^2} - \kappa^2\right)F_h(\kappa, z) = \sin(h\pi z) \tag{A.1}$$

with the boundary conditions

$$F_h = F_h' = 0, \quad z = 0, 1. \tag{A.2}$$

The solution is

$$F_h(\kappa, z) = A_h(\kappa)\sin(h\pi z) + C_h^{(1)}(\kappa) z\cos(\kappa z) + C_h^{(2)}(\kappa)\sin(\kappa z) + C_h^{(3)}(\kappa) z\sin(\kappa z), \tag{A.3}$$

where

$$\begin{aligned} A_h(\kappa) &= 1/(h^2\pi^2 + \kappa^2)^2 \\ C_h^{(2)}(\kappa) &= h\pi A_h(\kappa)/[(-1)^h \sin\kappa - \kappa] \\ C_h^{(1)}(\kappa) &= -\kappa C_h^{(2)}(\kappa) - h\pi A_h(\kappa) \\ C_h^{(3)}(\kappa) &= -C_h^{(2)}(\kappa) - \cos\kappa\, C_h^{(7)}(\kappa)/\sin\kappa. \end{aligned} \tag{A.4}$$

Definition of the Coefficients a, b *and* c

$$a(h, \nu, g) = \int_0^1 (2z - 1)F_h(\nu, z)\sin(g\pi z)\,dz \tag{A.5}$$

$$b(h, \kappa, f, g) = \int_0^1 F_h'(\kappa, z)\sin(f\pi z)\sin(g\pi z)\,dz \tag{A.6}$$

$$c(h, \kappa, f, g) = -f\pi\int_0^1 F_h(\kappa, z)\cos(f\pi z)\sin(g\pi z)\,dz. \tag{A.7}$$

CONVECTION THERMIQUE DANS UNE COUCHE HORIZONTALE DE FLUIDE AVEC SOURCES DE CHALEUR INTERNES

Résumé—Cet article concerne la convection thermique dans une couche horizontale de fluide limitée par deux plans rigides à températures constantes et égales. La convection est générée par des sources de chaleur internes uniformément distribuées. On trouve que des hexagones stables s'établissent pour des nombres de Rayleigh qui atteignent 3,6 fois la valeur critique, des hexagones dans un sens descendant quand le fluide est chauffé intérieurement, dans l'autre sens quand le fluide est refroidi. De plus, les hexagones peuvent exister dans une région sous-critique.

THERMISCHE KONVEKTION IN EINER WAAGRECHTEN FLUIDSCHICHT MIT INNEREN WÄRMEQUELLEN

Zusammenfassung—Diese Arbeit behandelt die thermische Konvektion in einer waagrechten Fluidschicht, die unten und oben von zwei starren Ebenen konstanter und gleicher Temperatur begrenzt wird. Die Konvektion wird erzeugt durch einheitlich verteilte innere Wärme-(Kälte-) Quellen. Stabile Sechskante stellten sich für Rayleighzahlen bis zum 3,6-fachen des kritischen Wertes ein; abwärtsgerichtete Sechskantströmungen, wenn das Fluid von innen beheizt wird und aufwärtsgerichtete, wenn das Fluid von innen gekühlt wird. Außerdem wurde ein überkritischer Bereich, in dem die Sechskante bestehen können, gefunden.

ТЕПЛОВАЯ КОНВЕКЦИЯ В ГОРИЗОНТАЛЬНОМ СЛОЕ ЖИДКОСТИ С ВНУТРЕННИМИ ИСТОЧНИКАМИ ТЕПЛА

Аннотация — Рассматривается тепловая конвекция в горизонтальном слое жидкости, ограниченном снизу и сверху твердыми границами при постоянной и одинаковой температуре. Конвекция создается равномерно распределенными источниками тепла (холода). Найдено, что устойчивой структурой конвекции являются шестиугольники для чисел Рэлея, которые в 3.6 раза превышают критическое значение. В случае разогрева жидкости в слое наблюдаются опускные шестиугольники, а в случае охлаждения жидкости, подъемные шестиугольники. Также определена критическая область, в которой могут существовать шестиугольники.